LETTERS

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Interaction between an Alfvén wave and a particle undergoing acceleration along a magnetic field

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The energy exchange between a plasma particle and an Alfvén wave propagating along a magnetic field is considered. It is shown that, in the presence of an accelerating force parallel to the background magnetic field, there exists a new channel for nonresonant energy transfer between the wave and the particle. © *2000 American Institute of Physics.* [S1070-664X(00)02301-6]

In space plasma physics, there are many important applications involving the interaction between charged particles and transverse electromagnetic waves propagating along a background magnetic field.

For example, Alfvén waves play a crucial role in the solar corona[1,2] and also when newborn ions are picked up by solar wind.[3,4] The origin of energetic particles in space plasmas is also commonly attributed to interaction with Alfvén waves.[5,6] Alfvén waves represent an important component of magnetosphere-ionosphere coupling.[7]

Most of the discussion in the literature about energy exchange between Alfvén wave and plasma particles involves cyclotron resonant interactions between waves and particles.[8–10] But nonresonant mechanisms of particle energization by Alfvén waves are also possible. Thus, in the presence of Alfvén waves, lower hybrid waves can be generated due to the streaming instability,[11] or, parametric interaction.[12] Recently, Wu *et al.* (1997) have discussed the possibility that Alfvén waves may accelerate particles even when the resonance condition is not satisfied. If the Alfvén wave is sufficiently strong, they find that ions can be accelerated on time scales of the order $\Omega^{-1}, (\Omega = eB_0/mc)$.

In the present paper, we discuss another mechanism for nonresonant $(\Omega \gg |\omega - kv_z|)$ acceleration of particles by Alfvén waves. We consider a plasma where particles experience a force parallel to the background field. In such a plasma, a low frequency Alfvén wave can lead to nonresonant energy exchange between particle and the wave.

To determine how a particle moving with acceleration along a uniform background magnetic field $\mathbf{B}_0 = (0,0,B_0)$ is affected by the presence of a wave, the wave is assumed to be circularly polarized, with electric field vector $\mathbf{E} = (E_x, E_y, 0)$ and magnetic field vector $\mathbf{B} = (B_x, B_y, 0)$. The wave with frequency ω and wave vector k propagates along $\mathbf{B}_0$. We assume that $\omega, |kv_z| \ll \Omega$ and the cyclotron interaction is inefficient. Here v_z is the particle velocity along $\mathbf{B}_0$.

A key element of the work is that the particle is subject to an external force along the z-axis, which gives rise to an acceleration g. The equations of motion are as follows:

$$\dot{v}_x = \frac{e}{m\omega}(\omega - kv_z)E_x + \Omega v_y,$$

$$\dot{v}_y = \frac{e}{m\omega}(\omega - kv_z)E_y - \Omega v_x, \tag{1}$$

$$\dot{v}_z = g + \frac{ek}{m\omega}\mathbf{v}_\perp \cdot \mathbf{E}_\perp. \tag{2}$$

Our primary interest is in the possible energy exchange between particle and wave. In view of this, we derive from (1) and (2) an expression for the rate at which the particle's kinetic energy $K = m(v_z^2 + v_x^2 + v_y^2)/2$ changes with time,

$$\frac{1}{m}\frac{dK}{dt} = g(gt + v_{z0}) + \frac{e}{m}\left[\frac{gk}{\omega}\int_{t_0}^{t}\mathbf{v}_\perp \cdot \mathbf{E}_\perp \, d\tau + \mathbf{v}_\perp \cdot \mathbf{E}_\perp\right]. \tag{3}$$

Equation (3) shows that, in order to consider the possible energy exchange between the wave and plasma particle, we need an expression for the rate at which the particle gains energy from the wave $[e(\mathbf{v}_\perp \cdot \mathbf{E}_\perp)/m]$.

A first integral of motion of Eqs. (1) and (2) is as follows:

$$\frac{m}{2}(v_\perp^2+(v_z-V_A)^2)=-mg(V_At-z)+\text{const.} \tag{4}$$

If $g=0$, the energy of the particle in the frame of reference moving with the wave is conserved. In the presence of a parallel force ($g\neq 0$), the energy exchange between the particle and wave is less constrained. The actual exchange depends on the dynamics of the interaction.

Eqs. (1) and (2) in variables $v_\pm=(v_x\pm v_y)/2$, $E_\pm=(E_x\pm iE_y)/2$ convert to

$$\dot{v}_\pm \pm i\Omega v_\pm=\frac{e}{m\omega}(\omega-kv_z)E_\pm, \tag{5}$$

$$\dot{v}_z=g+2\frac{ek}{m\omega}(v_+E_-+v_-E_+). \tag{6}$$

We can obtain a formal solution for $v_\pm$ in implicit form,

$$v_\pm=\frac{e}{2m}\int_{t_0}^{t}E_\perp(\tau)e^{\pm i(\Omega(\tau-t)+\theta(\tau))}\left(1-\frac{kv_z(\tau)}{\omega}\right)d\tau, \tag{7}$$

where the wave amplitude $E_\perp=\sqrt{E_x^2+E_y^2}$, its phase $\theta(t)=\omega t-kz(t)$, and particle phase $\phi(t)=\Omega t+\theta(t)$ satisfy

$$E_\pm=\frac{E_\perp(t)}{2}e^{\pm i(\omega t-kz(t))}=\frac{E_\perp(t)}{2}e^{\pm i\theta(t)}. \tag{8}$$

In Eq. (7) we omitted the term which depends on the initial transverse velocity and does not contribute to the wave particle energy exchange for times larger than gyroperiod.

To see how the particle responds to the low frequency wave [see Eq. (4)], we use Eqs. (7) and (8) to obtain an expression for the rate of work done on the particle by the wave,

$$\mathbf{E}_\perp\cdot\mathbf{v}_\perp=\frac{eE_\perp(t)}{m\omega}\int_{t_0}^{t}E_\perp(\tau)\theta'(\tau)\cos(\phi(\tau)-\phi(t))d\tau. \tag{9}$$

We assume that the electromagnetic field, as seen by the particle, is constrained in a particular way, namely that it is switched on adiabatically and $E_\perp=0$ for $t<-T$, $E_\perp=\text{const}$ for $t>0$. We also assume that the derivative of the electric field amplitude $E_\perp(t)$ along the trajectory of the particle is zero at time $t=-T$ when the electric field starts to build up, $(dE_\perp/dt)_{t=-T}=0$. These constraints mean that at some point z_0 in laboratory system of coordinates, the wave electric field amplitude changes more slowly than linearly as a function of $z-z_0$, e.g., $E_\perp\propto(z-z_0)^2$. The integral in Eq. (9) then is given by

$$\int_{-T}^{t}E_\perp(\tau)\theta'(\tau)\cos(\phi(\tau)-\phi(t))d\tau=\frac{E_\perp\Omega\theta''(t)}{(\phi'(t))^3}+\frac{\theta'(t)}{(\phi'(t))^2}\frac{d}{dt}E_\perp \tag{10}$$

As can be seen, each integration by parts introduces additional factors of order

$$\frac{\theta''}{(\Omega+\omega-kv_z)^2},\quad \frac{1}{\Omega^2}\frac{d^{n+2}E_\perp}{dt^{n+2}}\left[\frac{d^nE_\perp}{dt^n}\right]^{-1}. \tag{11}$$

Assuming that these factors are small, then (10) gives a formal expansion in these small parameters.

Constraint above is a restriction upon the way in which the wave field is switched on. The first term in (11) arises from changes in the phase, while the second term in (11) arises from changes in the amplitude of the wave as seen by the particle. Note that energy exchange between particle and wave occurs even though conditions are far from gyroresonance. As can be seen from (10), this energy exchange does not occur if the particle moves with constant velocity v_z.

The amount of energy exchange is determined by the work which the wave does on the particle,

$$\mathbf{v}_\perp\cdot\mathbf{E}_\perp=\frac{e}{m\omega}\left[\frac{E_\perp^2\Omega\theta''(t)}{(\phi'(t))^3}+\frac{(\theta'(t)}{2(\phi'(t))^2}\frac{d}{dt}E_\perp^2\right]. \tag{12}$$

The integral of Eq. (12) is:

$$\int_{-T}^{t}\mathbf{v}_\perp\cdot\mathbf{E}_\perp d\tau=\frac{e}{2m\omega}\left[\frac{E_\perp^2\theta'(t)}{(\phi'(t))^2}+\int_{-T}^{t}\frac{E_\perp^2(\tau)\theta''(\tau)}{(\phi'(\tau))^2}d\tau\right] \tag{13}$$

Equation (13) describes the changes which are induced in the parallel velocity v_z of the particle by the combined action of the wave and the external parallel force. The first term in Eq. (13) is not related to the external parallel force, but only to the electromagnetic wave. This term shows explicitly that the particle tends to move with the wave (''wave-surfing''). In order to estimate the second term in Eq. (13), we use explicitly the assumption of adiabatic switching. For the interval between $\tau=0$ and $\tau=t$, the wave amplitude $E_\perp$ is constant and

$$\int_{0}^{t}\frac{E_\perp^2(\tau)\theta''(\tau)}{(\Omega+\theta'(\tau))^2}d\tau=-\frac{E_\perp^2k(v_z(t)-v_z(0))}{(\Omega+\theta'(t))(\Omega+\theta'(0))}. \tag{14}$$

During the time interval between $t=-T$ and $t=0$, the wave field is growing from 0 to $E_\perp$ and

$$\int_{-T}^{0}\frac{E_\perp^2(\tau)\theta''(\tau)}{(\phi'(\tau))^2}d\tau=-\frac{E_\perp^2}{\phi'(0)}+\int_{-T}^{0}\frac{1}{\phi'(\tau)}\frac{dE_\perp^2}{d\tau}dt \tag{15}$$

In the limit $\theta'(\tau)=\theta'(0)$, the integral in (15) vanishes. We can obtain an upper bound to the integral $E_\perp^2/(\Omega+\theta'(-T))$, if we consider the opposite limit, $\theta'(\tau)=\theta'(-T)$. In what follows, we will use the upper bound.

Neglecting $\theta'(t)$ in comparison to Ω in Eq. (13),

$$\int_{-T}^{t}\mathbf{v}_\perp\cdot\mathbf{E}_\perp d\tau=\frac{e}{2m\omega}\frac{E_\perp^2(\omega-k(2v_z-v_z(-T)))}{\Omega^2}. \tag{16}$$

The terms in Eq. (3) can now be evaluated by combining Eqs. (12) and (16).

$$\frac{1}{m}\frac{dK}{dt}=g(gt+v_{z0})-\frac{e^2gk^2}{2m^2\omega^2\Omega^2}E_\perp^2 \times(V_A+2v_z(t)-v_z(-T))+\frac{e^2}{2m^2}\frac{\omega-kv_z}{\omega\Omega^2}\frac{d}{dt}E_\perp^2. \tag{17}$$

The last term in Eq. (17) arises from the changes in the wave field during the time period when switching is going on. This term is ponderomotive in nature, and is not present at times $t>0$ when the wave field $E_\perp=\text{const}$.

The remaining terms in Eq. (17) can be rearranged as

$$\frac{dK}{dt}=mg(gt+v_{z0})-mg(V_A+2v_z(t)-v_{z0})\frac{B_\perp^2}{2B^2}, \tag{18}$$

where $v_{z0}=v_z(-T)$ is the initial parallel velocity of the particle. Expression (18) represents the rate of kinetic energy exchange between the plasma particle and the Alfvén wave.

If the particle with initial kinetic energy K_0 moves from $z_0=0$ to z the particle total energy change is

$$K-K_0-mgz=-mV_Ag\frac{B_\perp^2}{B_0^2}\left[(t+T)-\frac{V_A-v_{z0}}{2g}\right]. \tag{19}$$

Therefore the final kinetic energy gain differs from the works produced by the accelerating force $m\mathbf{g}$.

The second term in the square brackets is due to the ponderomotive effect; the value of this term depends on the details of the total scenario of interaction. This term will be canceled out if the wave field is switched off at later times.

System (1–2) was solved numerically. Figure 1 illustrates how the kinetic energy of the particle is affected by the electromagnetic wave. The kinetic energy is normalized to $mV_A^2/2$, time is measured in gyroperiods, and normalized acceleration is $g/V_A\Omega=0.01$. In Fig. 2 we also present the numerically calculated value of the left-hand side of Eq. (19), obtained using Eqs. (1) and (2). As can be seen, calculated energy exchange is close to that described by an approximate solution (19) $0.01\times(0.3)^2t$.

The expression which we have derived for the nonresonant exchange of energy between wave and particle (18) has been obtained under certain restrictions. Our analysis takes into account only the lowest order nonlinear term in wave amplitude. Also the first term on the right-hand side of Eq. (2) must be larger than the second term. With the help of Eq. (12), this restriction can be written $B_\perp^2/B_0^2\ll1$. Second, for the sake of simplicity, we have restricted our analysis to time intervals during which $|\Omega|\gg|\omega-kv_z(t)|$. This condition excludes the resonance no matter what direction the velocity vector has. Therefore, the terms neglected in the expansion (12) have no poles. Third, the first condition in (11) requires that the external force is not too large and that the wave has a sufficiently long wavelength $kg/\Omega^2\ll1$.

Another restriction is that we have assumed adiabatic switching of the wave field. This is application dependent. A typical situation in which we need to consider wave switching would arise in the case where an Alfvén wave arrives into an initially quiet plasma; in such a case, the switching

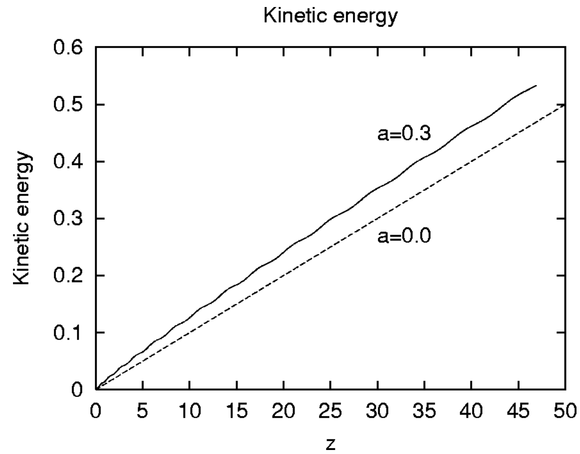

FIG. 1. Kinetic energy of the test particle for $a=B_\perp/B_0=0$ and 0.3 as function of height z.

time is determined by the ratio of the ramp width of the arriving pulse to $|v_z-V_A|$. This picture can be reversed and we may consider the particle entering a region where the Alfvén wave is present. In both cases the width of the transition region should be larger then the gyroradius of the particles. There will always be particles present which are so slow that they satisfy the adiabaticity condition. The condition for adiabatic switching can be written in terms of the switching time T as $T^2\Omega^2\gg1$.

According to Eq. (18) the particle kinetic energy increases with time if the $\mathbf{g}\cdot\mathbf{V}_A$ is negative. The particle increases its kinetic energy in situations where the external force is opposite to the direction of propagation of the wave. The rate of energy exchange is directly proportional to the wave velocity energy, its phase speed, and external force.

To clarify the role of the parallel force in the energization of the particle at the expense of the electromagnetic wave, we consider particle motion in the absence of parallel force and when wave amplitude is constant. The solution of Eqs. (1) and (2) for $r_\pm=(x\pm iy)/2$ is

$$r_\pm=\frac{eEe^{\pm\theta(t)}}{m\omega(\Omega+\omega-kv_{z0})},\quad v_z=v_{z0}. \tag{20}$$

The particle is rotating with the same angular frequency $\omega-kv_z$ as the electromagnetic field. Nevertheless, particle is not ''frozen in'' to the total magnetic field. The radius of the particle orbit is different from the radius of the cylinder on which the total magnetic field is wound. The particle velocity,

$$v_\pm=\pm i\frac{eE(\omega-kv_{z0})e^{\pm\theta(t)}}{m\omega(\Omega+\omega-kv_{z0})}$$

in this solution is orthogonal to the electric field of the wave. As a result, the wave work in Eqs. (3) is identically zero and the energy exchange between the particle and wave is inhibited. We also note that if the particle has different parallel velocity, its transverse velocity is also different. This means that when a parallel force is attempting to accelerate the particle along the z-axis, the wave electric field has to account for the change in v_z by also changing $v_\perp$. Indeed, the approximate solution of Eq. (7),

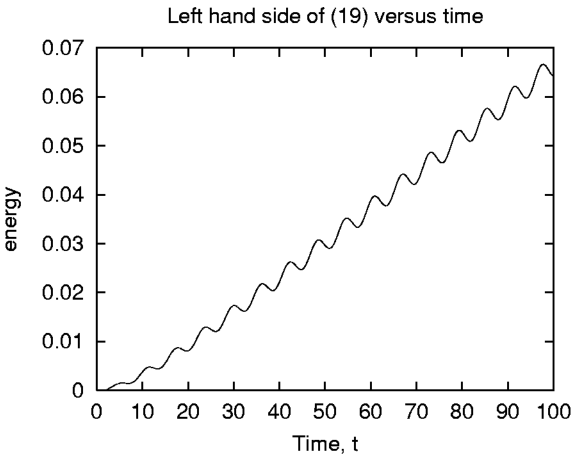

FIG. 2. The left-hand side of Eq. (19) as function of time calculated numerically for test particle motion.

$$v_{\pm}=\frac{e}{m\omega}\,\frac{e^{\pm i\theta(t)}}{(\Omega+\theta')}\left[\mp iE_{\perp}\theta'(t)-\left(\frac{E_{\perp}\theta'(t)}{\phi'(t)}\right)'_{t}\right] \tag{21}$$

shows that changes in v_z induce the additional component of particle velocity along the electric field. Particle is still ''precessing'' with the same angular velocity $\omega-kv_z$ as the electromagnetic field does, but now there is an additional phase shift in respect to the electric field. This shift builds up due to the accelerated motion of the particle and is proportional to $-kgt^2/2$. As a result, there is finite value of $(\mathbf{E}\cdot\mathbf{v})\propto -kg$ which induces the changes in both the transverse $K_{\perp}$ and parallel $K_{\parallel}$ parts of particle kinetic energy,

$$\frac{dK_{\perp}}{dt}=\frac{d}{dt}\left(\frac{mv_{\perp}^{2}}{2}\right)=e\left(1-\frac{kv_z}{\omega}\right)(\mathbf{E}\cdot\mathbf{v}), \tag{22}$$

$$\frac{dK_{\parallel}}{dt}=\frac{d}{dt}\left(\frac{mv_{z}^{2}}{2}\right)=mgv_z+e\frac{kv_z}{\omega}(\mathbf{E}\cdot\mathbf{v}). \tag{23}$$

The electromagnetic wave responds and provides additional acceleration to the particle so that to change the phase shift between wave induced transverse velocity of the particle and electric field back to 90°.

In this process, part of the gain in $v_{\perp}$ by the wave magnetic field is redirected to v_z as described by the terms proportional to kv_z/ω. This change in v_z is opposite to the change caused by the parallel force and provides the negative feedback to the particle motion from the wave. The change in total kinetic energy, however, is defined only by the first term in the right-hand side of (22), see also Eq. (3).

In the solar corona, collisional frequency is less than the particle gyrofrequency. The Alfvén waves propagating upward from the photosphere enter this region and can interact with the plasma via the mechanism described above. For estimates we use $B_0\approx 100$ G, $\Omega\approx 10^{7}$ rad s^{-1}, $\omega\approx 10^{-1}$ rad s^{-1}, $g=273$ m s^{-2}, and $B_{\perp}/B_0\approx 10^{-1}$. In hot, $T\approx 10^{6}$ K and dilute $n=10^{8}$ cm^{-3} solar corona, the collisional frequency is $\nu\approx 0.01$ s^{-1}. For particle velocity $v_{z0}\approx 1$ km s^{-1} and active region $\Delta z\approx 100$ km the acceleration time is $\approx 10^{2}$ s. Under these conditions the restrictions of our model are satisfied. The rate of energy deposition follows from (18),

$$n\frac{dK}{dt}\approx 3\times 10^{-9}\sqrt{n}B\frac{B_{\perp}^{2}}{B_{0}^{2}}\approx 3\times 10^{-3}\frac{B_{\perp}^{2}}{B_{0}^{2}}\ \text{ergs cm}^{-3}\ \text{s}^{-1},$$

which shows that proposed mechanism provides adequate heating rate for the solar corona.[1]

The above estimate demonstrates that the physical effect, which we have discussed in the report, can play a role as one of the possible mechanisms of solar corona heating. The essence of this mechanism is that the field aligned force serves as a mediator to provide energy exchange between waves and particles.

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